Basis risk

When speaking about forward or futures contracts, basis risk is the market risk mismatch between a position in the spot asset and the corresponding futures contract. More broadly speaking, basis risk (also called spread risk) is the market risk related to differences in the market performance of two similar positions. The more the instrument to hedged and the underlying used are imperfect substitutes, the bigger the basis risk is.

For example, a foreign exchange trader who is hedging a long spot position with a short forward position is taking the basis risk. While the spot position is sensitive only to changes in the exchange rate, the forward position is also affected by yield curve shifts. Accordingly, the two positions do not perfectly hedge one another, and the trader is taking basis risk.

A portfolio manager who wants to temporarily eliminate the market exposure of a diversified stock portfolio might short S&P 500 futures. If the composition of the portfolio does not exactly mirror the S&P 500, the hedge will not be perfect, and the portfolio manager will be taking basis risk. A swap trader who hedges herself/himself with bonds is also taking a basis risk.

Complex risks are sometimes described as consisting of several simpler risks, one or more of which are spread risks. For example, the market risk of corporate bonds can be described as comprising Treasury yield curve risk as well as the spread risk between Treasury yields and corporate yields.
The basis is traditionally defined as the difference between the futures price and the cash or street price. The basis is made up of a number of components: storage, interest, handling and transportation costs between the location and the futures delivery point, local supply and demand conditions, profit margins including opportunity costs, quanto and convexity correction (the latter is particularly relevant for interest rates futures). The nearby futures month is normally used to calculate the basis.

\[ \text{Basis} = \text{Futures} - \text{Spot} \]  \hspace{1cm} (1)

Factors that increase the basis are:

- Interest costs, storage costs, positive handling and transportation costs between the location and the futures delivery point
- Positive convexity correction as for Eurodollar futures. The convexity is positive in the case of negative correlation between the underlying of the futures contract and the interest rates.
- Positive quanto correction, in the case of positive correlation between the Foreign exchange rate used to compute the value of the underlying of the futures and the underlying of the futures itself.

While factors that decreases the basis are:

- Shortage of local supply on the spot market
- Positive dividends paid by the underlying asset of the futures contract
- Known positive cash flows generated by the underlying asset of the futures contract

The basis may be consistent over time but in certain situations it may fluctuate considerably. The basis risk concerns the risk associated with unexpected changes in the basis between the time a hedge is placed and the time that it is lifted. Unfortunately, hedging cannot eliminate basis risk.

Entering in a trade that speculates on the cost of carry is referred to as “basis trading”. It consists in taking the spread between the futures contract and the spot asset.

![Diagram of Futures Price in Contango and Backwardation](image)

**Figure 1**: Contango and backwardation for futures markets

When comparing the forward/futures prices with the spot price, one may find:
• A positive cost of carry, meaning that forward/futures prices $F$ are higher than spot prices $S$. The basis $F - S$ is positive. Futures markets are said to be in *contango*.

• A negative cost of carry, implying a negative basis. Futures markets are said to be in *backwardation*. This situation is also referred to as *inverted market*.

Figure 1 summarises the two situations and shows that at maturity futures converge to spot. At maturity, the basis converges to zero. (Neglecting some technical problem such as the wild card effect for bond futures)

In order to do basis trading, it is important to know how to compute the fair value of the basis. Long the basis means long futures and short the spot while short the basis is exactly the opposite trade.

Table 1, below gives example of the computation of the fair value of the futures contract. The notations used for the table are

• $r$ is the rate used to compute the cost of financing, close to the risk free rate as read from the standard interest rate curve and adjusted by the funding cost of the trading desk

• $q$ is the continuous yield dividend of the underlying asset of the futures contract

• $g$ is the yield of the storage and transportation costs also called the convenience yield for commodity futures
• \( I \) is the present value of the different cash flows generated by the underlying asset of the futures contract

• \( r_{\text{foreign}} \) is the foreign funding rate while \( r_{\text{dom}} = r \) is the domestic one

• \( \sigma_{HL} \) is the yearly Ho&Lee volatility (typical values are around 1%)

• \( T \) is the time to maturity of the futures contract

• \( T_{U} \) is the time to maturity of the rate underlying the futures contract

• \( \rho(FX, U) \) is the correlation between the FX rate \( X \) and the underlying asset \( U \) of the futures contract

• \( \rho(IR, U) \) is the correlation between the spot interest rate \( IR \) and the underlying asset \( U \) of the futures contract

• \( X \) is the rate used to compute the quanto futures. The futures pays \( S_{\text{foreign}} \) in domestic currency and \( X \) is domestic/foreign

• \( \sigma_{X} \) is the yearly volatility of the foreign rate

• \( \sigma_{S_{\text{foreign}}} \) is the volatility of the foreign asset underlying the futures contract

We then have the following table to compute the fair value of the futures. Real life examples often include a combination of the simple case describe in the table 1. (see Quantity-adjusting options (quantos))
<table>
<thead>
<tr>
<th>Case</th>
<th>Value of the Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow(s) with present value $I$</td>
<td>$(S_0 - I) e^{rT}$</td>
</tr>
<tr>
<td>Know continuous dividend yield $q$</td>
<td>$S_0 e^{(r-q)T}$</td>
</tr>
<tr>
<td>Storage cost $g$</td>
<td>$S_0 e^{(r+g)T}$</td>
</tr>
<tr>
<td>Foreign currency forward</td>
<td>$S_0 e^{(r_{dom} - r_{foreign})T}$</td>
</tr>
<tr>
<td>Convexity correction (this is added on top of the normal value of the forward)</td>
<td>$\text{Exp}(\frac{-1}{2} \sigma \rho (IR, U) \sigma_U T_U)$</td>
</tr>
<tr>
<td>Quanto correction (this is added on top of the normal value of the forward)</td>
<td>$\text{Exp}(\rho (X, U) \sigma_X \sigma_U T)$</td>
</tr>
</tbody>
</table>

Table 1: Example of computation of Futures in order to compute the fair value of the basis

Eric Benhamou

Swaps Strategy, London, FICC

Goldman Sachs International

\(^1\) The views and opinions expressed herein are the ones of the author’s and do not necessarily reflect those of Goldman Sachs