# Non-parametric option pricing models

The goal of *non-parametric option pricing models* is to price and risk mange financial derivatives in a model-free approach. Standard option pricing models need to assume a certain dynamics for the underlying. Model parameters are calibrated (or bootstrapped) to match certain conditions. These can be an exact fit to some market instruments whenever possible, a best fit otherwise, or some risk minimisation criterion. However, standard approaches do not eliminate model risk. What if the intrinsic model assumptions are flawed? What if the price dynamic is not a 1, 2 or 3 factor but much more? What if prices cannot be modelled by a diffusion model? Or if prices are not a continuous process?

Non-parametric option pricing methods *aims at eliminating model risk* by assuming no pre-specified model forms. Usually quite computer intensive, non-parametric option pricing methods have received more and more attention from researchers as computer performance has dramatically increased over the last years. In their most model-free approaches, non-parametric option pricing models assume as little as possible. They only rely on large amount of data on options and/or underlying prices to detect patterns and/or relationships on these inputs. In a non-parametric model, it is data that determine the dynamics of the underlying as well as pricing relationship.

Traditionally, non-parametric option pricing models are divided in two categories:

- **Rational non-parametric pricing models and theory**: these models use some financial assumptions to price and risk-manage derivatives. The following families of models fall into this category:
  - *Final distribution models* used widely for static replication. These models use the fact that the density of the underlying's price can be obtained by deriving twice the price of the options with respect to the strike (result of Litzenberger). The goal of the model is to get a very accurate estimation of the final distribution. Various smoothing methods, like, for instance, cubic-splines based ones, but also optimal fit to Laguerre, Legendre or various other polynomes can help obtain a regularised density.
  - *Generalised deterministic volatility models* assume that the volatility is a deterministic function of the underlying asset, which function is bootstrapped such as to match market prices on various call and put options with different strikes and maturities. One of the most famous methods is the one of Dupire (1993) using the forward PDE to calibrate the volatility surface to avoid the interpolation dilemma as well as possible arbitrage in the model.
  - *Equivalent martingale measure models* adjusting the drift for a preference-free risk neutral market. Under the risk-adjusted probability, the discounted[1] underlying asset has to be a martingale (a stochastic process whose best forecast of

[1] For equity, one has to take into account the dividends as these can be reinvested into the stock market. Similarly, for a futures contracts, the return is exactly equal to the risk-free rate, hence a driftless asset.

tomorrow's price is today's price). The model is non-parametric in the sense that the stochastic part of the dynamics is only restricted to be a diffusion and in a more general framework a jump diffusion or a Levy process. Kernel

  - *Non-parametric pricing models* using a combination of jumps and stochastic volatility where the distribution of jumps and the volatility is estimated by non-parametric methods.

- **Parameter-free models that do not rely on any financial theory for the interpolation of their data**. The common framework is to use the non-parametric model to give a forecast or estimate of a parameter of a given financial model. The most famous case is to use the non-parametric model to compute which volatility to use in the Black&Scholes model. Considered to be in a sense capturing the historical or equilibrium value dynamics of the model parameter, these non-parametric models are often used to do relative value trading. For the non-parametric algorithm to be efficient, it is often advocated to use high frequency data. Also, the estimation can use additional information like transaction volumes, transaction costs, and so on, to account for liquidity issues. As a matter of fact, there are several ways of estimating the model parameters:
  - *Genetic Programming algorithms*. Inspired by biology, these models tries various combinations of operators (like for instance moving averages, and the basic arithmetic operators), crossing them over (like genes), eliminating the worst performing rules

and mutating then again up to a convergence point. One of the variants of genetic programming is to represent the different rules as a tree and to use a heuristic walk over this tree, instead of random mutation, to find the best pricing method. This is in particular the method used by Keber (1998) to outperform the analytical approximations for American options.

- *Kernel regression methods*. These methods use a ‘smoother’ (a sophisticated process to average data to reduce error). Multi-variate and non-linear regressions are traditionally entering this category. Typical examples are the models of Hardle (1993). Ait-Sahalia et al. (1995). They suggested that kernel methods could provide more accurate pricing of an American option with stochastic dividends and stochastic volatility, than the regular analytical approximations.

- *Artificial Neural Networks (ANN) methods*. Quite popular some time ago, these computing methods strive at dynamically estimating model parameters by a learning process where the various weights given to each individual component of the network are changed during the learning process. These methods are in fact very similar to other filtering methods like Kalman filtering. There is an extensive literature on neural networks applied to finance (see for instance Malliaris and Salchenberger (1993), Hutchinson et al. (1994) Qi and Maddala

(1995), Hanke (1997) Anders et al. (1996), White (1989), Galindo (1998), Refenes (2000), Carelli (2000)).

The main problems of non-parametric pricing models are:

- Their appplicability and reliability rely heavily on the availability of a sufficient amount of reliable and representative data. High frequency data are often the best ones to use. This is why non-parametric models are often used to do relative trading on very liquid markets (hence lots of transaction data).
- Their poor performance in the case of a regime shift, although some models can account for this.
- The fact that they are disconnected from any financial reality, which can make them look like black boxes completely disconnected from the fundamentals except via the data. This means that the model can introduce some arbitrage in the pricing methodology. It is often wise to cross-check the output of these models.

On the other hand, one last point that can be argued to be their strength is that they are not influenced or corrupted by financial reasoning that may be quite inaccurate.

Entry category: options

Scope: Model free approach, volatility smile, kernel density estimation, artificial intelligence, neural network and genetic programming.

Related articles: options pricing models

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