Duration for callable securities
(effective duration)

Effective duration or duration for callable securities is the appropriate way to estimate the sensitivity of callable securities and more generally any bond and financial security with embedded option to the change of the interest rates term structure. In comparison with standard duration, effective duration takes into account not only complicated move the yield curve generated by a stochastic interest rate term structure model but also the change in value of the option due to the change of the underlying interest rates.

Used in asset liability management (*see Duration-based hedging/matching*), duration is an estimate of the sensitivity of a financial security or cash flow to interest rate changes. Properly applied, duration help to risk manage and hedge interest rate risk. However, standard concept of duration like Macaulay duration (*see Duration, modified (Macauley)*) or modified duration assumes:

- that cash flows do not change with interest rates. This is obviously wrong for Collateralized Mortgage Obligations (CMOs), Callable bonds, Loss reserves.
- that the yield term structure is flat. Generally yield curves are upward sloping.
- that a move of interest rate leads to a parallel shift in interest rates. This is also realistic, as short-term interest rates tend to be more volatile than longer-term rates.

Effective duration tends to answer to these criticisms. More precisely, effective duration:

- accommodates interest sensitive cash flows by taking into account the change of the option due to the change of the interest rates.
- can assume any type of yield term structure as it is based on a term structure model.
- allows for non-parallel interest rate shifts as the curve scenario generated are the one given by the term structure model used.

To compute the effective duration, one needs to adopt a term structure model. Standard choices used commonly by practitioners include

- the Vasicek and its extension to the Hull and White model, whose one factor version is given by the following diffusion on the short rate $dr(t) = (\alpha_t - \lambda_t r(t))dt + \sigma r(t) dW_t$ (*see Vasicek (1977) model*)
- the Cox, Ingersoll, Ross model, whose one factor version is given by the following diffusion on the short rate $dr(t) = (\alpha_t - \lambda_t r(t))dt + \sigma \sqrt{r(t)} dW_t$ (*see Cox-Ingersoll-Ross (CIR) interest rate model (1985)*)
- the market model or Brace Gatarek model, where the diffusion is described in terms of the Libor rate $\frac{dLibor(t,T,T+\Delta)}{Libor(t,T,T+\Delta)} = \sigma(t) dW_t^{T+\Delta}$ (see Brace, Gatarek & Musiela (1997) model (LIBOR market model))
- various multi-factor of the above models as well as factor models.

One computes therefore the price of the security before and after a bump of one of the meaningful parameters of the term structure model. Standard ways of pricing the embedded option are depending on the model but range from Monte Carlo simulations to the use of Partial Differential methods and trees.

Originally developed as a market risk measurement for bonds (the greater the duration or 'average' maturity, the greater the risk), effective duration has proven useful in analyzing equity securities and other fixed income options and futures.

Variations of effective duration are partial duration and option adjusted duration:

Partial duration consists in applying some of the principles of duration analysis to rate changes that affect only part of the yield curve, typically the shorter end of the curve. Partial durations will sum to a value that is usually close to the overall effective duration. Similar concepts are reshaping durations, and key-rate duration.

Option adjusted incorporates the expected duration-shortening effect of an issuer's embedded call provision and is also called adjusted duration.

Effective duration turns out to be very different than modified and Macaulay duration for securities with in the money embedded options and strong sensitivity to convexity.

Using the knowledge of the effective duration enables to hedge more appropriately the interest rate risk using liquid instruments such as government bonds and Forward Rate Agreement. For instance, if the effective duration of a callable bond is 5.8 while the duration of the 7 year government bond is of 5.1, to hedge 100 millions of notional of the callable bond requires 113 millions of notional of the 7 year government bond. Obviously, for cost of the balance sheet, the trading desk will enter a repo transaction to borrow 113 millions of notional of the 7 year government bond without depreciating too much its balance sheet.

See also duration, modified duration, convexity and immunisation.

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[1] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

References

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