Swaps: Constant maturity swaps (CMS) and constant maturity Treasury (CMT) swaps

A Constant Maturity Swap (CMS) swap is a swap where one of the legs pays (respectively receives) a swap rate of a fixed maturity, while the other leg receives (respectively pays) fixed (most common) or floating. A CMT swap is very similar to a CMS swap, with the exception that one pays the par yield of a Treasury bond, note or bill instead of the swap rate.

More generally, one calls Constant Maturity Swap and Constant Maturity Treasury derivatives, derivatives that refer to a swap rate of a given maturity or a pay yield of a bond, note or bill with a constant maturity. Since most likely, treasury issued on the market will not exactly match the maturity of the reference rate, one needs to interpolate market yield. (rates published by the British Banker Association in Europe and by the Federal Reserve Bank of New York)

MARKETING OF THESE PRODUCTS

CMT and CMS swaps provide a flexible and market efficient access to long dated interest rates. On the liability side, CMS and CMT swaps offer the ability to hedge long-dated positions. Great clients have been life insurers as they are heavily indebted in long dated payment obligations. Generous insurance policies need to be hedged against the sharp rise of the back end of the interest rate curve. Typical trade is a swap where they received the swap rate. On the asset side, corporate and other financial institutions have heavily invested in CMS market to enjoy yield enhancement and diversified funding. In a very steep curve environment, swaps paying CMS look very attractive to clients that think that the swap rates would not go as high as the market (and the forward curve) is pricing. Alternatively, in a flat yield curve environment, swaps receiving CMS look very attractive to market participants thinking that swap rates would rise in the futures as a consequence of the steepening of the curve. In a swap where one pays Libor plus a spread versus receiving CMS 10 year, the structure is mainly sensitive to the slope of the interest rate yield curve and is almost immunized against any parallel shift of the interest rate yield curve.

For all these reasons, it is not surprising that the CMS markets and the CMS options markets now trade in large quantities, both interbank and between corporates and financial institutions.

Pricing

Because of the increasing size of the CMS market, the market has seen its margin eroding. Banks have developed more and more advanced models to account for the smile, resulting in first a more pronounced smile and also an increasingly spread between CMS swap and their swaption hedge.

There exist two different methodologies for pricing CMS swaps:

- Parametric computation of the CMS convexity correction (See Hull(200), Benhamou (1999) and (2000)). In this approach, one assumes a model and uses some (smart) approximation methods to compute the expected swap rate under the forward measure. Non parametric computation of the swap rates. This approach assumes

- Non parametric computation of the CMS rates. This approach tries to minimize the amount of hypothesis between the computation of the CMS rate (see the works of Amblard, Lebuchoux (2000), Pugachevsky (2001)).

Note also that practitioners focus heavily on the computation of the forward CMS as they use these modified forwards and the volatility read from swaption market to compute simple options on CMS (CMS cap and floor, CMS swaption). This practice is justified by the fact that the first order effect comes mainly from the convexity corrected forwards as opposed to modified volatility assumptions. Using the same vol is therefore right at first order approximation, and strictly right in a Black Scholes setting.

Let use derive shortly the sketch lines of the two methods mentioned above. First, one can rapidly see that pricing a CMS swap boils down to price a simple swap rate received at time T. This can be done under the forward measure forward neutral measure Q_T, leading to compute:

$$E^{Q_T}\left[Sw(T, T_1, ..., T_n)\right], \tag{1.1}$$

where $E^{Q_T}[\ ]$ is the expectation under the forward neutral measure Q_T, and $Sw(T, T_1, ..., T_n)$ the value at time T of the swap rate with fixed payment dates $T_1, ..., T_n$.

We can then use standard change of numeraire technique to change the expression above. The natural numeraire for the swap rate is the annuity (also called level or dvo1, defined as the pv of one basis points paid over the life of the forward swap rate) of the swap rate, denoted by $LVL(T)$. This leads to:

$$E^{Q_T}\left[Sw(T,T_1,\ldots,T_n)\right]=E^{LVL_T}\left[\frac{B(T,T)}{LVL(T)}*\frac{LVL(0)}{B(0,T)}*Sw(T,T_1,\ldots,T_n)\right] \quad (1.2)$$

since $$\frac{dQ_T}{dQ^{LVL_T}}=\frac{B(T,T)}{LVL(T)}*\frac{LVL(0)}{B(0,T)}. \quad (1.3)$$

This shows that the CMS rate is equal to the swap rate plus an extra term function of the covariance under the annuity measure between the forward swap rate and the forward annuity:

$$\begin{aligned} &E^{Q_T}\left[Sw(T,T_1,\ldots,T_n)\right] \\ &= Sw(0,T_1,\ldots,T_n)+CovQ^{LVL_T}\left(\frac{LVL(0)B(T,T)}{LVL(T)B(0,T)},Sw(T,T_1,\ldots,T_n)\right) \end{aligned} \quad (1.4)$$

As a result, the CMS rate depends on the following three components:

- The yield curve via the swap rate and the annuity.
- The volatility of the forward annuity and the forward swap rate.
- The correlation between the forward annuity and the forward swap rate.

The first method relies on deriving an approximation for the covariance terms. There are many ways of doing this, in particular, using one factor approximation with lognormal assumptions, Wiener chaos expansion or simply martingale theory . To be more specific, let us examine the lognormal case. It assumes a lognormal martingale diffusion for the swap rate under the annuity measure:

$$\frac{dS(T,T_1,...,T_n)}{S(T,T_1,...,T_n)} = \sigma_t dW_t \tag{1.5}$$

The one factor approximation relies on assuming that the level can be represented as a function of the swap rate (which is rigorously true for cash settled swaptions). This leads to

$$LVL(T) = f\left(S(T,T_1,...,T_n)\right) \tag{1.6}$$

One can show that the adjustment is given by:

$$\begin{aligned} &CovQ^{LVL_T}\left(\frac{LVL(0)B(T,T)}{LVL(T)B(0,T)}, Sw(T,T_1,...,T_n)\right) \\ &\approx Sw(0,T_1,...,T_n)\frac{LVL(0)}{B(0,T)}\exp\left(-\frac{f'(Sw(0,T_1,...,T_n))}{f(Sw(0,T_1,...,T_n))}\sigma^2 Sw(0,T_1,...,T_n)T\right) \end{aligned} \tag{1.7}$$

The second approach relies on the fact that in the one factor approximation; the computation boils down to computing:

$$E^{Q^{LVL_T}}\left[\frac{Sw(T,T_1,...,T_n)}{f(Sw(0,T_1,...,T_n))}\right] \tag{1.8}$$

But we know that any function of only the swap rate can be evaluated as a portfolio of swaptions[1]. This comes from the fact that an expectation can be translated into an integral of the integrand times the density function of the swap rate. We can therefore evaluate the CMS swap rate as a portfolio of swaptions. As a matter of fact, replicating CMS with cash settled swaptions is accurate, while one needs to make a one factor approximation to extend the replication argument to physical settled swaptions. Using regression ideas, one can also extend the ideas of CMS replication to deferred payment CMS structures.

[1] see Breeden Litzenberger (1979) result on the fact that the second order derivatives of a call price with respect to the strike is simply the density function, hence the result

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Related articles: Swaps: complex structures; Swaps: taxonomy

[2] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

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